A STUDY OF SOLUTION STRATEGIES FOR SOME GRAPH EMBEDDING PROBLEMS

A Synopsis

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1. Introduction

Many problems of practical interest can easily be represented in the form of graph theoretical optimization problems like the Travelling Salesman Problem, Time Table Scheduling Problem etc. Recently, the application of metaheuristics and development of algorithms for problem solving has gained particular importance in the field of Computer Science and specially Graph Theory. Although various problems are polynomial time solvable, there are large number of problems which are NP-hard. Such problems can be dealt with using metaheuristics.

Metaheuristics are a successful alternative to classical ways of solving optimization problems, to provide satisfactory solutions to large and complex problems. Although they are alternative methods to address optimization problems, there is no theoretical guarantee on results [JJM] but usually provide near optimal solutions in practice. Using heuristic designs, metaheuristics attempt to escape the trap of local optimality by exploring the search space. Based on intensification (exploitation) and diversification (exploration) as well as on the iterative nature of the search space, metaheuristics can be defined as follows [OL]:

> "A metaheuristic is an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions."

Metaheuristics can be classified as population based (e.g., Genetic Algorithm etc.) and trajectory based (e.g. Tabu Search etc). Among population based methods, Evolutionary

Algorithms [KJ] are search methods that take inspiration from the biological world's natural selection and survival of the fittest. Evolutionary Algorithms are principally a stochastic search and optimization method [KJ]. Performing well on noisy functions in case of multiple local optima is one strength of Evolutionary Algorithms [KJ]. They do not ideally make any assumption about the fitness landscape,

so they perform well at approximating solutions to all types of problems. Stationary optimization problems are widely solved using Evolutionary Algorithms.

Metaheuristics have been used to solve optimization problems in graph theory and in particular graph layout problems which are discussed in section 3.

2. Evolutionary Algorithms

Evolutionary Algorithms, unlike traditional optimization, do not work on single point, instead they work on techniques involving entire populations of solutions. Each iteration in an Evolutionary Algorithm weeds out poor solutions by competitive selection, further finding the fitness for each solution for comparison. Some solutions are swapped (crossover), and single elements of the solution are changed (mutation) in order to find better solutions. Thus, we can say that they generate solutions to optimization problems by mimicking the process of natural evolution.

EAs are global optimization methods that guarantee convergence to a global optimum in a weak probabilistic sense. One of the strengths of EAs is that they perform well on perturbed functions where there are multiple local optima. EAs can often find a globally optimal solution without getting stuck on a local optima. They are characterized by the representation of the individual, the evaluation function representing the fitness level of the individuals, and the population dynamics such as population size, variation operators, parent selection, reproduction and inheritance, survival competition method, etc. [KJ].

The structure of evolutionary algorithms is as follows: [ZM]

```
begin
    t ← 0   initialize
    P(t) evaluate P(t)
    while (not termination condition) do
    begin     t ← t+1
              select P(t) from P(t-1)
             alter P(t)
```

evaluate P(t)

end

end

The Evolutionary Algorithm is a probabilistic algorithm which maintains the population of individuals, $P(t) = \{x_1(t), \ldots, x_n(t)\}$ for iteration t. Each individual represents the solution for the problem. Each solution $x_1(t)$ is evaluated to give some measure of fitness. Then a new population is formed (iteration t+1) by selecting the more fit individuals. Some members of the new population undergo transformation by means of genetic operators to form new solutions. After some number of generations, the program converges and it is hoped that best individual represents near optimal solution [ZM]. EAs are robust, global, and may be applied generally without recourse to domain specific heuristics, compared to traditional optimization methods.

3. Graph Layout Problems

Formally, a layout is an embedding of a graph G (V, E), where V and E are the set of vertices and edges of G respectively, onto a host graph H. It is a one-to-one mapping of the vertices of G with the vertices of H [JKA]. Mathematically, let G and H be graphs such that $|V(G)| = |V(H)|$. The embedding of G onto H is a pair of mappings (φ, ψ) where $\varphi: V(G) \rightarrow V(H)$ is a bijection and $\psi: E(G) \rightarrow \{$set of all paths in $H\}$, such that if $uv \in E$ then $\psi(uv)$ is the shortest path between $\varphi(u)$ and $\varphi(v)$ [JKA]. The set of all layouts of G is given by $\Phi(G)$ [JJM]. Two natural costs of a graph embedding are the dilation-cost and the expansion-cost [JKA].

Graph Layout Problems are a particular class of optimization problems, whose goal is to find a layout of an input in such a way that a certain objective is optimized. A large number of problems in different domains can be formulated as graph layout problems, including optimization of networks for parallel computer architectures, VLSI circuit design, information retrieval, numerical analysis, computational biology, graph theory, scheduling and archaeology [WPEA]. Most graph layout problems are NP-hard, but feasible solutions with an almost optimal cost are sufficient for their applications [JJM]. These problems have been approached using both evolutionary algorithms and approximation algorithms

[JJM, RB]. However, given the complexity of most problems approximation algorithms are still emerging and in many cases for special classes of graphs only [JJM].

3.1 Definitions

A linear layout, of an undirected graph $G = (V, E)$ with $n = |V|$ vertices is a bijective function $\varphi: V \rightarrow [n] = \{1, \ldots., n\}$. A layout can also be called a linear ordering, a linear arrangement, a numbering or a labelling of the vertices of a graph [JJM]. In other words, linear layout of G is the embedding of G onto the path graph H.

Given a layout φ of a graph $G = (V, E)$ and an integer i, we define the set

$$L(i, \varphi, G) = \{u \varepsilon V: \varphi(u) \leq i\} \text{ and the set } R(i, \varphi, G) = \{u \varepsilon V: \varphi(u) > i\}.$$

The edge cut at position i of φ is defined as

$$\theta(i, \varphi, G) = |\{uv \varepsilon E: u \varepsilon L(i, \varphi, G) \Lambda v \varepsilon R(i, \varphi, G)\}|$$

and the modified edge cut at position i of φ as

$$\varsigma(i, \varphi, G) = |\{uv \varepsilon E: u \varepsilon L(i, \varphi, G) \Lambda v \varepsilon R(i, \varphi, G) \Lambda \varphi(u) \neq i\}|.$$

The vertex cut or separation at position i of φ is defined as

$$\delta(i, \varphi, G) = |\{u \varepsilon L(i, \varphi, G) : \exists v \varepsilon R(i, \varphi, G) : uv \varepsilon E\}|.$$

Given a layout φ of G and an edge $uv \varepsilon E$, the length of uv on φ is

$$\lambda(uv, \varphi, G) = |\varphi(u) - \varphi(v)|.$$

3.2 Some Linear Graph Layout Problems:

Bandwidth (BW) Minimization Problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \varepsilon$ $\Phi(G)$ such that $BW(\varphi^*, G) = MINBW(G)$, where $BW(\varphi, G) = \max_{uv \epsilon E} \lambda(uv, \varphi, G)$.

Minimum Linear Arrangement (LA): Given a graph $G = (V, E)$, find a layout $\varphi^* \varepsilon \Phi(G)$ such that $LA(\varphi^*, G) = MINLA(G)$, where $LA(\varphi, G) = \sum_{uv \epsilon E} \lambda(uv, \varphi, G)$.

Cutwidth (CW) Minimization Problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \varepsilon \Phi(G)$ such that $CW(\varphi^*, G) = MINCW(G)$, where $CW(\varphi, G) = \max_{i \epsilon [|V|]} \theta(i, \varphi, G)$.

Modified Cut (MC) Minimization Problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $MC(\varphi^*, G) = MINMC(G)$, where $MC(\varphi, G) = \sum_{i \in [|V|]} \varsigma(i, \varphi, G)$.

Vertex Separation (VS) or Pathwidth (PW) Minimization Problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $VS(\varphi^*, G) = MINVS(G)$, where $VS(\varphi, G) = \max_{i \in [|V|]} \delta(i, \varphi, G)$.

Edge Bisection (EB) Minimization Problem: Given a graph G = (V, E), find a layout $\varphi^* \in \Phi(G)$ such that $EB(\varphi^*, G) = MINEB(G)$, where $EB(\varphi, G) = \theta(\lfloor \frac{1}{2} |V| \rfloor, \varphi, G)$.

Antibandwidth (ABW) Maximization Problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $ABW(\varphi^*, G) = MAXABW(G)$, where $ABW(\varphi, G) = \min_{uv \in E} \lambda(uv, \varphi, G)$.

3.3 Cyclic Layout Problems:

When a graph G is embedded onto a cycle then it is known as cyclic embedding and the corresponding problems are termed as cyclic layout problems. Some of these problems are defined in this section.

Cyclic Antibandwidth maximization problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $CAB(\varphi^*, G) = MAXCAB(G)$, where $CAB(\varphi, G) = \min_{uv \in E} \lambda_C(uv, \varphi)$.

Cyclic Bandwidth sum minimization problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $CBS(\varphi^*, G) = MINCBS(G)$, where $CBS(\varphi, G) = \sum_{uv \in E} \lambda_C(uv, \varphi)$.

Cyclic Bandwidth minimization problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $CBW(\varphi^*, G) = MINCBW(G)$, where $CBW(\varphi, G) = \max_{uv \in E} \lambda_C(uv, \varphi)$.

Cyclic Cutwidth minimization problem: Given a graph $G = (V, E)$, find a layout $\varphi^* \in \Phi(G)$ such that $CCW(\varphi^*, G) = MINCCW(G)$, where $CCW(\varphi, G) = cg(G, C_n)$, where C_n is a cycle with n vertices, $cg = |\{f \in E : e \in \psi(f)\}|$.

3.4 Short Literature Review of Metaheuristics and Exact results for some

Graph Layout Problems

The exact results of Bandwidth Minimization Problem (BMP) for path, cycle, bipartite graphs [YK], triangulated graphs, are available in literature [RCM]. Some metaheuristics have also been proposed in the literature for the problem. A Wonder Bandwidth Reduction Algorithm [AMFL] and a Tabu Search (TS) was originally proposed by Cuthill-McKee [EJ]. Pinana et al. [EIVR] have used a Greedy Randomised Adaptive Search Procedure (GRASP) with path relinking. An integrated hill climbing procedure with Genetic Algorithm was proposed by Lim et al. [ABFa]. They also implemented a hybrid heuristic to solve the BMP[ABFb]. Mladenovic et al. [NDDC] have proposed a Variable Neighbourhood Search (VNS). A branch and bound algorithm has also been proposed for linear and cyclic bandwidth problem [HEG]. Tabu Search and Simulated Annealing have been applied for cyclic bandwidth minimization problem [EHGF].

The Cutwidth Minimization Problem (CMP) deals with the number of edges passing through a vertex when all vertices are arranged on line, cycle or tree as host graph. It was first used as a theoretical model in an optimal layout of a circuit. The exact cutwidth of a path, cycle and complete graph by Chung [FLR], complete bipartite graphs by Liu et al. [YJa], hypercube by Yixun et al. [LLY] are found in literature. A few metaheuristics have been proposed to solve CMP. Andrade [DM] proposed GRASP with path relinking which starts with a random depth first search (DFS). A scatter search has been developed by Pantrigo et al. [JRAE]. Bansal et al. [RKS] have used multipoint crossover and intermediate depth first search on some standard classes of graphs. A variable neighbourhood search strategy for this problem has been proposed by Parado et al. [ENJA]. Diaz et al. [JJM], have identified a class of graphs for which the problem is polynomial time solvable. Memetic algorithms have also been applied to a diversity of problems [JCA, RCJF, RJLY, SRA, AW]. For Cyclic Cutwidth Minimization Problem a memetic algorithm has been proposed in which six construction heuristics have been designed [PKG]. It has also been proved that cyclic cutwidth of tress is equal to linear cutwidth [JR].

Minimum linear arrangement problem (MinLA) is NP-complete in general [JPa]. MinLA has applications in graph drawing [FOLI] and single machine job scheduling [DLA, RAP]. The exact results for MinLA for path, cycle, star and bipartite graphs [YK], Hypercube and complete binary tree are known in literature. It is one of the most studied GLP. A useful construction heuristic based on the computation of fielder vector G was proposed. Petit [JPb] proposed some algorithms like successive augmentation heuristics, local search heuristics etc. Koren and Harel [YD] proposed a multiscale algorithm. Tello et al. [EJJ] generated an MA using Front Incremental Method algorithm. Safro et al. [IDA] developed a new multilevel algorithm for general graph. Pantrigo et al. [JAVR] developed some other heuristics for the problem. Sharma et al. [RK] designed a hybrid EA for this problem. Marti et al. [RMFV] have proposed a Scatter Search and path relinking. The Cyclic Bandwidth Sum minimization problem is the cyclic case of MinLA problem. The exact results of star and wheel [HJ], path cycle, complete binary tree, complete bipartite graphs [YJb] are found in literature. A general Variable Neighbourhood Search is developed by Satsangi et al. [DKGb].

3.5 Graph Layout on a Grid

Graph layout problems, when the host graph is a grid, arise in the formulation of circuit layout models for VLSI design and optimization. A k-dimensional grid graph G_i has a vertex- set consisting of k-tuples of integers, and edge set consisting of $\{(a_1 \ldots\ldots\ldots, a_i, \ldots\ldots a_k), (a_1 \ldots\ldots\ldots, a_i + 1, \ldots\ldots a_k)\} \forall a_i$ and $\forall i$ [JKA]. For a two dimensional grid graph with two vertices (a,b) and (c,d), the distance is defined as $d_H = |a\text{-}c| + |b\text{-}d|$. Parameters to access the quality of an embedding of graph G over H, are congestion (cutwidth in case of linear and cyclic layout), the maximal number of paths that share $e \in H$, dilation (bandwidth in case of linear and cyclic layout), the length of the largest associated path and the load i.e. maximum number of $u \in V(G)$ that are mapped to H. Mathematically, Congestion of an edge $e \in E(H)$ under the embedding (φ, ψ) of G onto H is defined as $cg(G, H, \varphi, \psi, e) = |\{ f \in E : e \in \psi(f)\}|$. The congestion of G in H under (φ, ψ) is $cg(G, H, \varphi, \psi) = \max_{e \in H} \{cg(G, H, \varphi, \psi, e)\}$ and the congestion of G in H is defined as $cg(G, H) = \min_{(\varphi, \psi)} \{cg(G, H, \varphi, \psi, e)\}$ [PKG]. Dilation of an edge $e \in E(H)$ under the embedding (φ, ψ) of G onto H is defined as $B(G, H) = \min_{\varphi \in \psi} |\varphi|$, where $|\varphi| = \max_{uv \in G} d_H$

$(\varphi(x), \varphi(y))$ [CZ]. Chung [FLR], had proved lower and upper bounds with host graph as a grid. However, this problem remains unstudied from the point of view of metaheuristics and exact methods. Also, no work was found in the surveyed literature when the host graph is a tree.

In graph layout problems, there are objective functions like cutwidth, bandwidth, bandwidth sum, vertex separation, etc., with optimization problems associated with each. Also, a lot of literature can be found with host graphs being paths and cycles, with various applications of each problem independently. Diaz et.al, [JJM] have given inequalities as direct consequences of definitions of linear graph layout problems, like $\mathrm{LA}(G) \leq n \,.\, \mathrm{CW}(G)$, $\mathrm{LA}(G) \leq m \,.\, \mathrm{BW}(G)$, etc. [JJM], where n is order, and m is the size of G. However, it remains an open problem to explore the effect of change in one objective over another. For instance, an attempt to optimize the bandwidth may adversely affect the cutwidth of the layout and vice versa. Many other such pairs may be found. Such relations on other pair of problems need to be explored.

4. Page Number Problem

When the layout of a graph G is linear and the edges are placed on more than one plane so that they do not intersect, such an embedding is known as the book embedding of graph, since the line can be considered as the spine of a book and the planes as the pages of book. The page number of a graph is the thickness of the smallest (in number of pages) book into which G can be embedded. Formally, the page number of a graph G is the minimum number of pages of the book into which G can be embedded without crossing of edges and this problem is known as the page numbering problem. This problem has applications in layout problems arising in interconnecting networks, single row routing, VLSI design, etc. [NMI]. The exact results of page number of cycle, star, bipartite graphs and triangulated graphs have been given by Swaminathan et al. [RDD]. Chung et al. [FFA] have given results on complete graphs, pinwheel graphs, hypercubes, cube connected cycle, complete binary tree and planar graphs. Kapoor et al. [NMI] gave the first metaheuristic using a GA. Later, Satsangi et al. [DKGa] proposed a DFS based solution for the problem. The book embedding may be extended to the case wherein the

vertices are laid out on a grid, i.e. the host graph is a grid. From the metaheuristics point of view not much work has been done in the literature surveyed by us so far.

5. Application to Learning Analytics

A large number of problems in different domains can be formulated as graph layout problems, including optimization of networks for parallel computer architectures, VLSI circuit design, information retrieval, numerical analysis, computational biology, graph theory, scheduling and archaeology [WPEA]. Learning Analytics is also one such area wherein graph based models can be developed and studied.

Analytics is the extensive use of data to analyse, statistically and quantitatively, to develop predictive and exploratory models and draw insights to aid decision making. The field of analytics has gained much importance in the recent years, especially in the context of big data [AMM].

Learning Analytics, as adopted by the First International conference on Learning Analytics & Knowledge [LAK11], is defined as "the measurement, collection, analysis and reporting of data about learners and their contexts, for purposes of understanding and optimizing learning and the environments in which it occurs". As an emerging field, it is helping in devising new methods to analyse educational data through the development of generic frameworks [WH, DSG].

Given that many problems in learning analytics can be modelled using graphs, a study of the application of graph layouts to the domain of learning analytics can be carried out as this has not been reported in the literature.

6. Objectives

1. To explore and identify specific applications of metaheuristics for cutwidth minimization problem and bandwidth minimization problem, when the host graph is a grid/tree.
2. To study the relationships and effect between different graph layout problems in the case of a linear graph layout.

3. To explore and design metaheuristics for page number minimization problem when the host graph is a grid.
4. To develop a graph based framework for mapping programmes, programme pathways and programme dependencies in an University with the objective to
    a Map graph layout problems to a graph instance defined through the framework.
    b Identify different properties related to programmes in a University vis-à-vis graph layout properties.

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